Final Report

Analytical Analysis of Vehicle Routing and Inventory Routing Problems
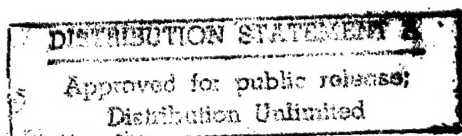
Award No. N00014-90-J-1649

1. Research on Vehicle Routing and Related Models

The objective of the project is to perform **analytical analyses** of heuristics for the Vehicle Routing Problem (VRP) and apply the results in models that integrate vehicle routing with other issues important to the firm. Below, we highlight our major accomplishments during that period.

A. Determined the exact structure of the asymptotic optimal solution value of the VRP with capacity and time window constraints and used it to develop a new and efficient algorithm. In Bramel and Simchi-Levi (1996) we have characterized the asymptotic optimal cost of the VRP with Capacity and Time Window constraints. This characterization enabled us to understand the algorithmic structure required to solve large size VRPs and consequently to develop efficient algorithms for the VRP with capacity and time window constraints. Computational experiments show that these algorithms outperform many published heuristics on standard test problems, see Bramel and Simchi-Levi (1996).

B. Obtained a characterization of the effectiveness of set partitioning formulations for VRPs. A classical and recently popular technique that has proven effective for solving vehicle routing problems is based on formulating them as a Set Partitioning Problem. An empirically observed property is that the optimal solution values of the Set Partitioning Problem and its linear programming relaxation are very close. In Bramel and Simchi-Levi (1997), we explain this behavior by proving that for any distribution of the service times, time windows, customer loads and locations, the relative gap between fractional and integer solutions of the Set Partitioning Problem becomes arbitrarily small as the number of customers increases.

19990108 011

COLUMBIA UNIVERSITY

IN THE CITY OF NEW YORK

OFFICE OF PROJECTS AND GRANTS

December 30, 1998
OPG: 8762

Marc J. Lipman, Scientific Officer
Office of Naval Research
ATTN: Code 1111 SP
Ballston Tower One
800 North Quincy Street
Arlington, VA 22217-5000

RE: *Final Patent Report Submission for ONR Award No. N00014-90-J-1649*

Dear Dr. Lipman:

In accordance with the terms and conditions of the referenced grant, we enclose three (3) copies of the Final Technical Report and one (1) copy of the completed Final Patent/Invention Report (form DD-882) for the award entitled " Analytical Analysis of Vehicle Routing and Inventory Routing Problems." The report covers the period of May 1, 1990 to September 30, 1995.

Additional copies of the report have been forwarded as indicated below.

In addition, with respect to the New Technology/Patent Rights clause of the referenced grant, we wish to certify that to our knowledge there were no reportable items developed under this award.

If you have any questions, please contact me at (212) 854-6851 or via e-mail at: <alw10@columbia.edu>.

Sincerely,

Aleta Walker-Boddie

Aleta Walker - Boddie
SRO Projects Assistant

Encl.

Cc: Angela Potter, Administrative Contracting Officer {w/cy FTR + original and 2 cy DD-882}
    Department of the Navy
    Office of Naval Research   Mr. William McCarthy {w/cy DD-882 only}
    495 Summer Street – 6 th Floor -- Room 627   ONR/Ballston Tower One/ATTN: ONR OOCCI
    Boston, MA 02210-2109   800 North Quincy Street; Arlington, VA 22217-5660

    Director, Naval Research Laboratory {w/ 2 cy}   Defense Technical Information Center {w/ 2cy}
    ATTN: Code 5227   8725 John J. Kingman Road STE 0944
    Washington, DC 20375-5326   Ft. Belvoir, VA 22060-6218

    File

C. Characterized the worst-case behavior of the linear programming relaxation of the set partitioning formulation for the bin-packing problem. Many researchers have reported that the optimal solution values of the linear programming relaxation of the set partitioning formulation of a variety of combinatorial problems is quite close to the optimal integer solution of that problem, even for **small size** problems. This observation motivated us to characterize, in Chan, Simchi-Levi and Bramel (1998), the maximum deviation between the linear programming relaxation of the set-partitioning formulation of the bin-packing problem and the optimal solution to the problem. A by-product of our analysis is an **improved worst-case** bound for the classical first-fit decreasing and best-fit decreasing heuristics.

D. Performed a probabilistic analysis of a multi-echelon distribution system and used the insight obtained to develop an efficient algorithm for it. In Chan and Simchi-Levi (1994) we analyzed the problem of integrating inventory control and vehicle routing into a cost-effective strategy for a distribution system consisting of a single outside vendor, a fixed number of warehouses and many geographically dispersed retailers. The results and insights obtained have also been used to develop new tools that assist the design and operations of such systems. These results generalize earlier results obtained in Chan, Federgruen and Simchi-Levi (1998) for a single echelon model.

Many of these results are summarized in a survey paper, see Bertsimas and Simchi-Levi (1996), that is going to appear in *Operations Research.* We are also pleased to note that Lap Mui Ann Chan, a Ph.D student supported by the project and working on these problems under the supervision of the PI, was awarded an **honorable mention** in the **1994 ORSA Nicholson Student Paper Competition** and the **1995 INFORMS Nicholson Student Paper Competition**. She joins Julien Bramel, a former Ph.D. student of the PI who was also supported by the project, in receiving these prestigious awards.

2. References

[1] Bertsimas, D. and D. Simchi-Levi (1996) The New Generation of Vehicle Routing Research: Robust Algorithms Addressing Uncertainty. *Operations Research*, **44**, pp. 286–304.

[2] Bramel J. and D. Simchi-Levi (1997) On the Effectiveness of Set-Covering Formulations for the Vehicle Routing Problem. *Operations Research*, **45**, pp. 295–301.

[3] Bramel, J. and D. Simchi-Levi (1996), Probabilistic analysis and Practical Algorithms for the Vehicle Routing Problem with Time Windows. *Operations Research*, **44**, pp. 501–509.

[4] Chan, L. M. A., A. Federgruen and D. Simchi-Levi (1998), Probabilistic Analysis and Practical Algorithms for Inventory Routing Models. *Operations Research*, **46**, pp. 96–106.

[5] Chan, L. M. A. and D. Simchi-Levi (1994), Probabilistic Analysis and Practical Algorithms for Three-Level Distribution Systems. To appear in *Management Science*.

[6] Chan, L. M. A., D. Simchi-Levi and J. Bramel (1998), Worst-case Analyses, Linear Programming and the Bin-packing Problem. *Mathematical Programming*, **83**, pp. 213–227.

3. Publications

Below we provide a list of refereed publications that acknowledge the support of ONR contract N00014-90-J-1649

[1] Anily, S., J. Bramel and D. Simchi-Levi (1994), Worst-case Analysis of Heuristics for the Bin-Packing Problem with General Cost Structure. *Operations Research* **42**, 287–297.

[2] Berman, O., P. Jaillet and D. Simchi-Levi (1995). Recent Developments in Location-Routing Problems with Uncertainty. *Facilities Location*, Z. Drezner edt., Springer Verlag, pp. 427–373.

[3] Bertsimas, D. and D. Simchi-Levi (1996) The New Generation of Vehicle Routing Research: Robust Algorithms Addressing Uncertainty. *Operations Research*, **44**, pp.

286–304.

[4] Bienstock, D., Bramel, J. and D. Simchi-Levi (1993), A Probabilistic Analysis of Tour Partitioning Heuristics for the Capacitated Vehicle Routing Problem with Unsplit Demands. *Mathematics of Operations Research* **18**, 786–802.

[5] Bienstock, D., M. Goemans, D. Simchi-Levi and D. Williamson (1993), A Note on the Prize Collecting Traveling Salesman Problem. *Mathematical Programming* **59**, pp. 413–420.

[6] Bramel, J., E. G. Coffman Jr., P. W. Shor and D. Simchi-Levi (1992) Probabilistic Analysis of the Capacitated Vehicle Routing Problem with Unsplit Demands. *Operations Research* **40**, pp. 1095–1106.

[7] Bramel, J., C. L. Li and D. Simchi-Levi (1993), Probabilistic analysis of Heuristics for the Vehicle Routing Problem with Time Windows. *American Journal of Mathematical and Management Sciences*, **13**, pp. 267-322.

[8] Bramel, J., W. T. Rhee and D. Simchi-Levi (1997), Average Case Analysis of the Bin-Packing Problem with General Cost Structure. *Naval Research Logistics*, **44**, pp. 673-686.

[9] Bramel, J. and D. Simchi-Levi (1995), A Location Based Heuristic for General Routing Problems. *Operations research* **43**, pp. 649–660.

[10] Bramel J. and D. Simchi-Levi (1997) On the Effectiveness of Set-Covering Formulations for the Vehicle Routing Problem. *Operations Research*, **45**, pp. 295–301.

[11] Bramel, J. and D. Simchi-Levi (1996), Probabilistic analysis and Practical Algorithms for the Vehicle Routing Problem with Time Windows. *Operations Research*, **44**, pp. 501–509.

[12] Chan, L. M. A., A. Federgruen and D. Simchi-Levi (1998), Probabilistic Analysis and Practical Algorithms for Inventory Routing Models. *Operations Research*, **46**, pp. 96–106.

[13] Chan, L. M. A. and D. Simchi-Levi (1994), Probabilistic Analysis and Practical Algo-

rithms for Three-Level Distribution Systems. To appear in *Management Science.*

[14] Chan, L. M. A., D. Simchi-Levi and J. Bramel (1998), Worst-case Analyses, Linear Programming and the Bin-packing Problem. *Mathematical Programming*, **83**, pp. 213–227.

[15] Federgruen, A. and D. Simchi-Levi (1995), Analytical Analysis of Vehicle Routing and Inventory-Routing Problems. *Handbooks in Operations Research and Management Science,* the volume on Networks and Distributions edited by M. Ball, T. Magnanti, C. Monma and G. Nemhauser, North-Holland, Amsterdam, pp. 297–373.

[16] Gallego, G., D. Simchi-Levi and D. Xiao (1992), The Complexity of the Staggering Problem and other Classical Inventory Problems. *Operations Research Letters* **12**, pp. 47–52.

[17] Gallego, G., M. Queyranne and D. Simchi-Levi (1996), Single Resource Multi-Item Inventory System. *Operations Research*, **44**, pp. 580–595.

[18] Gavish, B., C. L. Li and D. Simchi-Levi (1992), Analysis of Heuristics for the Design of Tree Networks. *Annals of Operations Research* **36**, pp. 77–86.

[19] Li, C. L., D. Simchi-Levi and M. Desrochers (1992), On the Distance Constrained Vehicle Routing Problem. *Operations Research* **40**, pp. 790–800.

[20] Li, C. L., S. T. McCormick and D. Simchi-Levi (1992), On the Minimum-Cardinality-Bounded Diameter and the Bounded-Cardinality-Minimum Diameter Edge Addition Problems. *Operations Research Letters* **11**, pp. 303–308.

[21] Simchi-Levi, D. (1992), Hierarchical Planning for Probabilistic Distribution Systems. *Management Science* **38**, pp. 198–211.

[22] Simchi-Levi, D. (1994), New Worst Case Results for the Bin-Packing Problem. *Naval Research Logistics*, **41**, pp. 579-585.